УДК 621.396.967 DOI: 10.31673/2786-8362.2024.011111

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A MATHEMATICAL REPRESENTATION OF AN AUTOMATED SYSTEM DESIGNED TO MONITOR CRITICAL INFRASTRUCTURE OBJECTS USING SEISMIC AND ACOUSTIC SIGNALS

Shevchenko A. A Mathematical representation of an automated system designed to monitor critical infrastructure objects using seismic and acoustic signals. The article considers the mathematical model of the automated system of seismoacoustic monitoring of critical infrastructure objects. In the monitoring approach, the object is identified with a point in the multidimensional space of free model parameters. Thus, the forecast about the state of the object is a forecast of a significant shift in the position of the parameter vector in the parameters space. When choosing a mathematical model, it is necessary to select a space of informative parameters to reduce the probability of errors of two types. First of all, it is necessary to build a mathematical model of the dynamics of the OKI, which reflects the most significant moments of the monitoring process, including both the process itself and the disturbances accompanying this process and the noise background superimposed on the process of the dynamics of the state of the OKI. A priori knowledge of the interference of an arbitrary process will significantly weaken its influence on obtaining estimates of the parameters of the process, which is perceived as a useful signal. This attenuation is achieved by optimizing processing procedures that take into account the a priori statistics of the random interference process. A new mathematical model in the form of a superposition of Berlasi pulses in the seismoacoustic frequency range and constructive algorithms for its implementation are proposed. The mathematical properties of the model were studied. Thus, the state of OKI is reflected in the vector of free parameters of the above model. To evaluate the informative parameters of the proposed model of the automated seismo-acoustic monitoring system, the work solves the problem of nonlinear regression, considering them as the point of the criterion optimum in the n-dimensional space. In the situation that has developed in our country, related to the conduct of military operations on its territory and missile attacks, the creation of automated seismo-acoustic monitoring systems of critical infrastructure objects is a necessary task.

Keywords: monitoring, critical infrastructure, acoustic signal, information parameters, automated system

Шевченко А. Математична модель автоматизованої системи сейсмоакустичного моніторингу об'єктів критичної інфраструктури. В статті розглядається математична модель автоматизованої системи сейсмоакустичного моніторигу обєктів критичної інфраструктури. У моніторинговому підході об'єкт ототожнюється з точкою у багатовимірному просторі вільних параметрів моделі. Таким чином, прогноз про стан об'єкта є прогнозом істотного зміщення положення вектора параметрів у просторі ознак.

Запропоновано нову математичну модель у вигляді суперпозиції імпульсів Берлазі, у сейсмоакустичному діапазоні частот, та конструктивні алгоритми її реалізації. Досліджено математичні властивості моделі. Таким чином стан обєкта критичної інфраструктури (ОКІ) відображається у векторі вільних параметрів вищевказаної моделі. Для оцінки інформативних параметрів запропонованої моделі автоматизованої системи сейсмоакустичного моніторингу у роботі вирішується завдання нелінійної регресії, розглядаючи їх як точку оптимуму критерію в n-мірному просторі. У ситуації, що склалася в нашій країні, пов'язаної з проведенням військових дій на її території, та ракетних атак створення автоматизованих систем сейсмоакустичного моніторингу обєктів критичної інфраструктури являється необхідною задачею.

Ключові слова: моніторинг, критична інфраструктура, акустичний сигнал, інформаційні параметри, автоматизована система

Introduction

This article is focused on the problem of detecting changes in the informative parameters of critical infrastructure objects (CIO) in order to assess the dynamics of its behavior against the background of microseismic noise. It's important to recognize that the seismic background noise emanating from critical infrastructure objects (CIO) imposes a natural constraint on the detectability of seismic equipment.

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The seminal research on detecting faint seismic signals within environments of high noise [1] provides an illustration where the characteristics of the signal to be detected are identified. However, detecting any signals in the recorded data, even under reduced noise conditions, would prove challenging without prior information about the event.

The advent of instrumental seismology has brought about a heightened focus on seismic noise, prompting closer examination and study. It was noted that seismic noise exhibits complexity, with its components possessing varied characteristics. Even in the present day, there remains a lack of consensus regarding the precise physical nature of this noise.

Furthermore, it is assumed [2] that the distinctive parameters observed during seismic and acoustic monitoring reflect the underlying structure of the critical infrastructure object (CIO). Diverse sources of natural noise were taken into account, including factors such as wind, vehicular traffic, and industrial activity disturbances. A sophisticated examination of the energy dependence of seismic background on frequency is detailed in [3].

In this study, we adopt a background model in which the seismic background is assumed to be energetically stationary within a relatively narrow spectral band. We will focus on the cyclic fluctuations of the background during periods devoid of industrial activity, which reflects the daily people's activities [4]. We disregard instrumental interference, deeming it insignificant compared to the levels of background values recorded.

The objective of detection processes is to scrutinize the disparity between signals and noise to enhance detection capabilities. Various detection methods can be employed depending on the type of seismic signals being analyzed. We will consider detection processes which operate on a single recording channel. For a comprehensive overview of this field, we can refer to studies on signal processing [5, 6]. The mathematical modeling approaches discussed in this article are centered on automated systems designed to monitor the dynamics of critical infrastructure object (CIO) behavior.

Procedures relying on the ratio of short-term signal power to long-term signal power have been employed for an extended period: detection is triggered if the ratio surpasses a specified threshold [7]. This approach can be utilized in the mathematical modeling within this paper to pre-estimate one of the parameters that determine the signal, specifically the timing of its occurrence.

The commonly employed filtering methodology relies on information regarding the ratio of noise to signal. The spectral components of seismic signals typically differ from those of noise, thus comparing spectral components is suggested as a potential method for achieving optimal filtering. In scenarios where the waveform of the signal to be detected is already identified, so-called waveform filters can be employed [1]. This technique can be perceived as a correlation between the detected signal and the main signal with a known shape. The main signal can be either a synthetic signal or a pre-recorded signal, as exemplified in [8, 9]. Noise-reducing filters can be useful in cases where only the signal type is known. These filters can be highly advantageous for estimating signal waveforms while mitigating the impact of noise interference. Some noise-reducing filters estimate noise based on its past behavior and subsequently remove anticipated noise [10]. However, these filters necessitate a high level of noise stability. Short-period noise may be regarded as relatively constant, at least within a small observation interval. Nevertheless, they typically exhibit lower effectiveness compared to conventional band filters [11]. Another type of noise-reducing filter is the so-called cut-off filter, which essentially eliminates noise peaks from the frequency band [12]. In noise reduction, the process involves eliminating from the data a hypothetical model of the microseismic background using optimal values of free parameters derived from the process's history [13, 14]. Subsequently, the optimal estimation of the received signal against the background of the noise, suppressed in this manner, is calculated. In this study, we introduce a model aimed at separating the useful signal from the noise.

Mathematical model. In this study, we exclusively examine physically feasible signals, namely those that adhere to wave conditions. [15]. This implies signals that meet two conditions: causality and stability, which are:

$$S(t,\mathbf{h})=\begin{cases}S(t,\mathbf{h}),t\geq 0\\0,t<0\end{cases};$$

$$\int_0^\infty \left(S(t,\mathbf{h})\right)^2 dt<\infty .$$

Where $\mathbf{h}$ represents the vector of free parameters constituting the signal, and t denotes time.

A physically realizable signal can be effectively represented as a superposition of oscillators. However, for actual physical systems characterized by finite spectral bandwidth and consequently, delayed wavefronts, it is more appropriate to model them as a superposition of Berlage pulses, extending the concept of oscillators, as illustrated in the expression:

$$y(t)=\eta(t)te^{-\alpha t}\sin(\omega t). \quad (1)$$

The subsequent extension is a natural progression for the model (2.1):

$$y(t)=A\eta(t-\tau)(t-\tau)\exp\{-\alpha(t-\tau)\}\sin\left[\omega(t-\tau)\right] \quad (2)$$

where $\eta(t-\tau)$ represents the Heaviside function for the Berlage impulse occurring at a specific time τ . The subsequent generalization of the model (generalized Berlage function):

$$y(t)=A\eta(t-\tau)(t-\tau)^{\beta}\exp\{-\alpha(t-\tau)\}\sin\left[\omega(t-\tau)\right] \quad (3)$$

Formulas 1-3 naturally extend the oscillator model, wherein:

- $y(t)$ - observed data,
- $\{A,\tau,\alpha,\omega,\beta\}$ vector of free parameters of the model, where A denotes the amplitude of oscillations,

τ - time of signal reception,

α - parameter characterizes the signal fading (decrement),

ω - signal frequency,

β - is a parameter that characterizes the signal front.

It is logical to opt for the Berlage pulse superposition model [5] as a parametric mathematical representation of the CIO. The free parameters of this model effectively describe the spectral characteristics of the CIO, enabling assessment of the object's behavior dynamics to mitigate potential adverse effects following the impact of a shock wave.

Estimation of the parameters of the mathematical model for the automated system of seismic and acoustic monitoring of critical infrastructure objects.

A model comprising K submodels, representing a superposition of pulses, each defined by formula (2.3) and incorporated into the model via a vector of physically meaningful free parameters of the connected Berlage pulse. Each pulse is fully determined by a vector-string:

$$\mathbf{P}_{\langle k\rangle}=\{A_k,\tau_k,\alpha_k,\omega_k,\beta_k\} \quad (4)$$

And thus, the model is taking shape:

$$M(t,\mathbf{P})=\sum_{k=1}^{K}A_k\eta(t-\tau_k)(t-\tau_k)^{\beta_k}\exp\{-\alpha_k(t-\tau_k)\}\sin\left[\omega_k(t-\tau_k)\right]+n(t) \quad (5)$$

The matrix of free parameters of the model is introduced in equation (2.5) $\mathbf{P}$ The row vector of this matrix fully defines the submodel, while the column vector defines the associated parameters of the submodels, including additive interference. $\mathbf{P}_{\langle k\rangle}$ k $\mathbf{P}^{\langle s\rangle}$ $n(t)$ There are five such related parameters in equation (5):

$$\mathbf{P}=\{P_{k,s}\},\ k=\overline{1,K},\ s=\overline{1,S}. \quad (6)$$

S - the number of free parameters in the submodel corresponds to the number of columns in the matrix $\mathbf{P}$

K - number of submodels corresponds to the number of strings in the matrix. $\mathbf{P}$

Therefore, the rectangular matrix of free parameters of the model $\mathbf{P}$ dimension $K \times S$ (4) defines the model (5).

If you combine related vectors into a matrix, then

$$\mathbf{P} = \left\{ \mathbf{P}^{\langle 1 \rangle}, \mathbf{P}^{\langle 2 \rangle}, \mathbf{P}^{\langle 3 \rangle}, \mathbf{P}^{\langle 4 \rangle}, \mathbf{P}^{\langle 5 \rangle} \right\}, \quad (6\text{-b})$$

In equation (2.6), combined vectors of the form are expressed as:

$$\mathbf{P}^{\langle 1 \rangle} = \mathbf{A} = \begin{Bmatrix} A_1 \\ \vdots \\ A_k \\ \vdots \\ A_K \end{Bmatrix}; \mathbf{P}^{\langle 2 \rangle} = \boldsymbol{\tau} = \begin{Bmatrix} \tau_1 \\ \vdots \\ \tau_k \\ \vdots \\ \tau_K \end{Bmatrix}; \mathbf{P}^{\langle 3 \rangle} = \boldsymbol{\alpha} = \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_k \\ \vdots \\ \alpha_K \end{Bmatrix};$$

$$\mathbf{P}^{\langle 4 \rangle} = \boldsymbol{\omega} = \begin{Bmatrix} \omega_1 \\ \vdots \\ \omega_k \\ \vdots \\ \omega_K \end{Bmatrix}; \mathbf{P}^{\langle 5 \rangle} = \boldsymbol{\beta} = \begin{Bmatrix} \beta_1 \\ \vdots \\ \beta_k \\ \vdots \\ \beta_K \end{Bmatrix} \quad (6\text{-c})$$

The model for optimizing the dynamic monitoring parameters of critical infrastructure objects based on seismic and acoustic analysis, in terms of the free parameter matrix, is presented as follows: $\mathbf{P}$

$$M(t,\mathbf{P}) = \sum_{k=1}^{K} \mathbf{P}_k^{\langle 1 \rangle} \eta(t - \mathbf{P}_k^{\langle 2 \rangle})(t - \mathbf{P}_k^{\langle 2 \rangle})^{\mathbf{P}_k^{\langle 5 \rangle}} \exp\left\{ -\mathbf{P}_k^{\langle 3 \rangle}(t - \mathbf{P}_k^{\langle 2 \rangle}) \right\} \sin\left[\mathbf{P}_k^{\langle 4 \rangle}(t - \mathbf{P}_k^{\langle 2 \rangle}) \right] \quad (6\text{-d})$$

Here, vectors $\mathbf{A}, \boldsymbol{\tau}, \boldsymbol{\alpha}, \boldsymbol{\omega}, \boldsymbol{\beta}$ are column vectors composed of K rows, where K represents the number of submodels. The matrix $\mathbf{P}$ is constructed as a combination of vectors (6), $P_{k,s} = P_k^{\langle s \rangle}$ In the provided model. $s = \overline{1,S}; S = 5.$

SInce the metric $L_2(0,T)$ represents the energy of the examined process, it is suitable to use it for constructing a criterion (target function) $Q(\mathbf{P})$ based on the model (6) and field observations $V(t)$ within the interval $(0,T)$ in the given metric. $L_2(0,T)$ $T = \Delta t \cdot S1$ where Δt represents the time quantum, and $S1$ denotes the number of sample points on the interval, which can be expressed by the following expressions:

$$Q(\mathbf{P}) = \left\| V_{s1} - M(t_{s1}, \mathbf{P}) \right\|_{L_2}; k = \overline{1,K} \quad (7)$$

$$Q(\mathbf{P}) = \sqrt{\frac{1}{T} \sum_{s1=0}^{S1} (V_{s1} - M(t_{s1}, \mathbf{P}))^2}; t_{s1} = s1 \cdot \Delta t; T = t_S \cdot \Delta t. \quad (7\text{-a})$$

To determine the extrema of the function $Q(\mathbf{P})$, it is necessary to solve the system of equations for all elements of the matrix: $\mathbf{P}$

$$\frac{\partial Q(\mathbf{P})}{\partial P_{s,k}} = 0, \; k = \overline{1,K}; s = \overline{1,S} \quad (8)$$

Considering the similarity of related submodels, we can consistently consider the variables in the direction of the vectors $\mathbf{P}^{\langle k \rangle}$, of related parameters of different submodels.

$$\frac{\partial Q(\mathbf{P})}{\partial \mathbf{P}^{\langle k\rangle}}=\begin{Bmatrix}\dfrac{\partial Q(\mathbf{P})}{\partial \mathbf{P}_1^{\langle k\rangle}}\\ \cdots \\ \dfrac{\partial Q(\mathbf{P})}{\partial \mathbf{P}_S^{\langle k\rangle}}\end{Bmatrix}=\mathbf{0}=\begin{Bmatrix}0\\ \cdots\\ 0\end{Bmatrix}.\ k=\overline{1,K}. \qquad (8)$$

The relevance of the variational approach in solving geophysical problems can be observed, for example, in [16].

The search for the minimum is carried out using the Leavenberg-Marquardt algorithm [17], for an a priori randomly selected point in the space of free parameters of the model (5)

Of the many extremums, a global minimum must be selected.

$$t\in(t_{s1_0},T_p)\ M(t,\mathbf{P})=\sum_{k=1}^{K}\mathbf{P}_k^{\langle 1\rangle}\eta(t-\mathbf{P}_k^{\langle 2\rangle})(t-\mathbf{P}_k^{\langle 2\rangle})^{\mathbf{P}_k^{\langle 5\rangle}}\exp\left\{-\mathbf{P}_k^{\langle 3\rangle}(t-\mathbf{P}_k^{\langle 2\rangle})\right\}\sin\left[\mathbf{P}_k^{\langle 4\rangle}(t-\mathbf{P}_k^{\langle 2\rangle})\right];$$

$t\in(t_{s1_0},T_p)$. For example, in the case of $\mathbf{P}_k^{\langle 5\rangle}=1$ *для* $\forall\ k$, i.e., when the superposition of Berlage pulses is assessed, the model derivative in the direction of the amplitude vector, i.e., the vector $\mathbf{P}^{\langle 1\rangle}$, will be:

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 1\rangle}}=\eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\sin\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right];k=\overline{1,K} \qquad (11)$$

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 1\rangle}}=\begin{Bmatrix}\eta(t-P_1^{\langle 2\rangle})(t-P_1^{\langle 2\rangle})\exp\left\{-P_1^{\langle 3\rangle}(t-P_1^{\langle 2\rangle})\right\}\sin\left[P_1^{\langle 4\rangle}(t-P_1^{\langle 2\rangle})\right]\\ \vdots\\ \eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\sin\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right]\\ \vdots\\ \eta(t-P_K^{\langle 2\rangle})(t-P_K^{\langle 2\rangle})\exp\left\{-P_K^{\langle 3\rangle}(t-P_K^{\langle 2\rangle})\right\}\sin\left[P_K^{\langle 4\rangle}(t-P_K^{\langle 2\rangle})\right]\end{Bmatrix} \qquad (11)$$

In the direction of the frequency vector $\mathbf{P}^{\langle 4\rangle}$.

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 4\rangle}}=P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\cos\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right]\ ;k=\overline{1,K} \qquad (12)$$

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 4\rangle}}=\begin{Bmatrix}P_1^{\langle 1\rangle}\eta(t-P_1^{\langle 2\rangle})(t-P_1^{\langle 2\rangle})\exp\left\{-P_1^{\langle 3\rangle}(t-P_1^{\langle 2\rangle})\right\}\cos\left[P_1^{\langle 4\rangle}(t-P_1^{\langle 2\rangle})\right]\\ \vdots\\ P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\cos\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right]\\ \vdots\\ P_K^{\langle 1\rangle}\eta(t-P_K^{\langle 2\rangle})(t-P_K^{\langle 2\rangle})\exp\left\{-P_K^{\langle 3\rangle}(t-P_K^{\langle 2\rangle})\right\}\cos\left[P_K^{\langle 4\rangle}(t-P_K^{\langle 2\rangle})\right]\end{Bmatrix} \qquad (12a)$$

In the direction of decrement vectors $\mathbf{P}^{\langle 3\rangle}$.

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 3\rangle}}=-P_k^{\langle 3\rangle}P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\sin\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right];k=\overline{1,K} \qquad (13)$$

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 3\rangle}} = \begin{Bmatrix} -P_1^{\langle 3\rangle}P_1^{\langle 1\rangle}\eta(t-P_1^{\langle 2\rangle})(t-P_1^{\langle 2\rangle})\exp\left\{-P_1^{\langle 3\rangle}(t-P_1^{\langle 2\rangle})\right\}\sin\left[P_1^{\langle 4\rangle}(t-P_1^{\langle 2\rangle})\right] \\ \vdots \\ -P_k^{\langle 3\rangle}P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})(t-P_k^{\langle 2\rangle})\exp\left\{-P_k^{\langle 3\rangle}(t-P_k^{\langle 2\rangle})\right\}\sin\left[P_k^{\langle 4\rangle}(t-P_k^{\langle 2\rangle})\right] \\ \vdots \\ -P_K^{\langle 3\rangle}P_K^{\langle 1\rangle}\eta(t-P_K^{\langle 2\rangle})(t-P_K^{\langle 2\rangle})\exp\left\{-P_K^{\langle 3\rangle}(t-P_K^{\langle 2\rangle})\right\}\sin\left[P_K^{\langle 4\rangle}(t-P_K^{\langle 2\rangle})\right] \end{Bmatrix} \quad (13a)$$

In the direction of the shift vector in the Heaviside function $\mathbf{P}^{\langle 2\rangle}$, the derivative in the direction of the signal reception moments vector τ is calculated, i.e. $\mathbf{P}^{\langle 2\rangle}$. The most complicated parameter is the moment of signal entry (τ), since $\mathbf{P}^{\langle 2\rangle}$ includes the generalized Heaviside function.

The derivative in the direction of the vector of entry moments τ , i.e. $\mathbf{P}^{\langle 2\rangle}$:

$$\begin{aligned} \frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 2\rangle}} &= P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\sin\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right]- \\ &-P_k^{\langle 1\rangle}\delta(t-P_k^{\langle 2\rangle})(P_k^{\langle 2\rangle}-t)\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\sin\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right]+ \\ &+P_k^{\langle 1\rangle}P_k^{\langle 4\rangle}\eta(t-P_k^{\langle 2\rangle})(P_k^{\langle 2\rangle}-t)\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\cos\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right]+ \\ &+P_k^{\langle 1\rangle}P_k^{\langle 3\rangle}\eta(t-P_k^{\langle 2\rangle})(P_k^{\langle 2\rangle}-t)\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\sin\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right];\ k=\overline{1,K} \end{aligned} \quad (14)$$

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 2\rangle}} = \begin{Bmatrix} P_1^{\langle 1\rangle}\eta(t-P_1^{\langle 2\rangle})\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\sin\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right]- \\ -P_1^{\langle 1\rangle}\delta(t-P_1^{\langle 2\rangle})(P_1^{\langle 2\rangle}-t)\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\sin\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right]+ \\ +P_1^{\langle 1\rangle}P_1^{\langle 4\rangle}\eta(t-P_1^{\langle 2\rangle})(P_1^{\langle 2\rangle}-t)\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\cos\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right]+ \\ +P_1^{\langle 1\rangle}P_1^{\langle 3\rangle}\eta(t-P_1^{\langle 2\rangle})(P_1^{\langle 2\rangle}-t)\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\sin\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right] \\ \cdots \\ P_K^{\langle 1\rangle}\eta(t-P_K^{\langle 2\rangle})\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\sin\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right]- \\ -P_K^{\langle 1\rangle}\delta(t-P_K^{\langle 2\rangle})(P_K^{\langle 2\rangle}-t)\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\sin\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right]+ \\ +P_K^{\langle 1\rangle}P_K^{\langle 4\rangle}\eta(t-P_K^{\langle 2\rangle})(P_K^{\langle 2\rangle}-t)\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\cos\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right]+ \\ +P_K^{\langle 1\rangle}P_K^{\langle 3\rangle}\eta(t-P_K^{\langle 2\rangle})(P_K^{\langle 2\rangle}-t)\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\sin\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right] \end{Bmatrix} \quad (14a)$$

We can see that the second element of this expression can be neglected, because the function $\delta(t-P^{\langle 2\rangle})$, in the entire range of existence, except for the point $P^{\langle 2\rangle}$, is 0, then:

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 2\rangle}} = P_k^{\langle 1\rangle}\eta(t-P_k^{\langle 2\rangle})\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\sin\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right]+$$
$$+P_k^{\langle 1\rangle}P_k^{\langle 4\rangle}\eta(t-P_k^{\langle 2\rangle})(P_k^{\langle 2\rangle}-t)\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\cos\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right]+$$
$$+P_k^{\langle 1\rangle}P_k^{\langle 3\rangle}\eta(t-P_k^{\langle 2\rangle})(P_k^{\langle 2\rangle}-t)\exp\left\{P_k^{\langle 3\rangle}(P_k^{\langle 2\rangle}-t)\right\}\sin\left[P_k^{\langle 4\rangle}(P_k^{\langle 2\rangle}-t)\right] \quad (15)$$

$$\frac{\partial M(t,\mathbf{P})}{\partial \mathbf{P}^{\langle 2\rangle}} = \left\{\begin{matrix} P_1^{\langle 1\rangle}\eta(t-P_1^{\langle 2\rangle})\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\sin\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right]+ \\ +P_1^{\langle 1\rangle}P_1^{\langle 4\rangle}\eta(t-P_1^{\langle 2\rangle})(P_1^{\langle 2\rangle}-t)\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\cos\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right]+ \\ +P_1^{\langle 1\rangle}P_1^{\langle 3\rangle}\eta(t-P_1^{\langle 2\rangle})(P_1^{\langle 2\rangle}-t)\exp\left\{P_1^{\langle 3\rangle}(P_1^{\langle 2\rangle}-t)\right\}\sin\left[P_1^{\langle 4\rangle}(P_1^{\langle 2\rangle}-t)\right] \\ \cdots \\ P_K^{\langle 1\rangle}\eta(t-P_K^{\langle 2\rangle})\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\sin\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right]+ \\ +P_K^{\langle 1\rangle}P_K^{\langle 4\rangle}\eta(t-P_K^{\langle 2\rangle})(P_K^{\langle 2\rangle}-t)\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\cos\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right]+ \\ +P_K^{\langle 1\rangle}P_K^{\langle 3\rangle}\eta(t-P_K^{\langle 2\rangle})(P_K^{\langle 2\rangle}-t)\exp\left\{P_K^{\langle 3\rangle}(P_K^{\langle 2\rangle}-t)\right\}\sin\left[P_K^{\langle 4\rangle}(P_K^{\langle 2\rangle}-t)\right] \end{matrix}\right\} \quad (15a)$$

The set of equations for the components of the matrix P will be as follows

$$\frac{\partial Q(P)}{\partial P^{\langle k\rangle}} = \sum_{s1=0}^{S1}\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}\left(V_{s1}-M(t_{s1},\mathbf{P})\right)^2 = 2\sum_{s1=0}^{S1}\left(V_{s1}-M(t_{s1},\mathbf{P})\right)\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P}) = 0 \Rightarrow$$
$$\Rightarrow \sum_{s1=0}^{S1}V_{s1}\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P}) - \sum_{s1=0}^{S1}M(t_{s1},\mathbf{P})\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P}) = \mathbf{0};\ k=\overline{1,K} \Rightarrow \quad (16)$$
$$\Rightarrow \sum_{s1=0}^{S1}M(t_{s1},\mathbf{P})\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P}) = \sum_{s1=0}^{S1}V_{s1}\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P});\quad k=\overline{1,K}$$

Where $\frac{\partial}{\partial \mathbf{P}^{\langle k\rangle}}M(t_{s1},\mathbf{P})$ is determined by formulas (12) - (15).

In the system of equations for free parameters, we highlight groups associated with derivatives in the direction of related parameters of submodels, which are combined into derivatives in the direction of the vector of these parameters, in which the components are ordered ascending by the parameter *k*. For example, a vector of amplitude parameters

$$P^{\langle 1\rangle} = \left\{\begin{matrix} P_1^{\langle 1\rangle} \\ \vdots \\ P_k^{\langle 1\rangle}\end{matrix}\right\} = \left\{\begin{matrix} A_1 \\ \vdots \\ A_k\end{matrix}\right\}$$

I. Group of equations related to derivatives in $P_{k1,}\ k=\overline{1,K}$:

$$\sum_{s=1}^{S1}V_{s,1}\eta(t_{s,1}-P_{k,2})[t_{s,1}-P_{k,2}]\exp\left\{-P_{k,3}(t_{s,1}-P_{k,2})\right\}\sin\left[P_{k,4}(t_{s,1}-P_{k,2})\right]-$$

$$-\sum_{s=1}^{S1}\sum_{\sigma=1}^{\Sigma}\eta(t_{s,1}-P_{\sigma,2})\eta(t_{s,1}-P_{k,2})[t_{s,1}-P_{\sigma,2}][t_{s,1}-P_{k,2}]\exp\left\{-P_{\sigma,3}(t_{s,1}-P_{\sigma,2})-P_{k,3}(t_{s,1}-P_{k,2})\right\}$$

$$\sin\left[P_{\sigma,4}(t_{s,1}-P_{\sigma,2})\right]\sin\left[P_{k,4}(t_{s,1}-P_{k,2})\right]=0\,;k=\overline{1K}\Rightarrow$$

$$\Rightarrow\sum_{s=1}^{S1}\sum_{\sigma=1}^{\Sigma}\eta(t_{s,1}-P_{\sigma,2})\eta(t_{s,1}-P_{k,2})[t_{s,1}-P_{\sigma,2}][t_{s,1}-P_{k,2}]\exp\left\{-P_{\sigma,3}(t_{s,1}-P_{\sigma,2})-P_{k,3}(t_{s,1}-P_{k,2})\right\}\times$$

$$\sin\left[P_{\sigma,4}(t_{s,1}-P_{\sigma,2})\right]\sin\left[P_{k,4}(t_{s,1}-P_{k,2})\right]=$$

$$=\sum_{s=1}^{S1}V_{s,1}\eta(t_{s,1}-P_{k,2})[t_{s,1}-P_{k,2}]\exp\left\{-P_{k,3}(t_{s,1}-P_{k,2})\right\}\sin\left[P_{k,4}(t_{s,1}-P_{k,2})\right];\quad k=\overline{1K}$$

Considering that the function

$$\eta(t_{s,1}-P_{k,2})=\begin{cases}0, & \Delta ts1\le P_{k,2}\\ 1, & \Delta ts1>P_{k,2}\end{cases},$$

then the summation of s_1 should start with $S_1^*=\dfrac{P_{\sigma,2}}{\Delta t}$ (naturally an integer part: Antje)

If $S_1^*=\dfrac{P_{\sigma,2}}{\Delta t}$ (Antje), then $\eta(t_{s,1}-P_{\sigma,2})\eta(t_{s,1}-P_{k,2})>0\quad npu\; t_{s1}>\max(s1_\sigma^*,s1_k^{**})$.

Problem (7) is a nonlinear regression problem. To solve it, it is necessary to calculate the global minimum of the criterion (7). To do this, we choose a set of points in the vicinity of which we find the local minimum of criterion (7) (in this problem, the set of points was selected by the Monte Carlo method, according to a priori distributions, so we get a point in the model parameter space in the vicinity of which there is a local extremum and look for a global one that gives the minimum of criterion (7) on the set of all local extremums, that is, we choose the smallest one on the set of all local minimums. From the set of local minimums, a point is selected that gives the minimum of the criterion minimum from a given set of points. This point is chosen as the solution.

We have ensured that this point is likely to converge to a rational solution in the sense of our criterion (7) with increasing growth [18].

Therefore, we find the global minimum of the functional $Q(\mathbf{P}_k)$ on the set of acceptable vectors $\mathbf{P}_k$ from the set A_k with a priori known distribution.

$$\min_{\mathbf{P}_k}\left\{Q(\mathbf{P}_k)\right\};\,k=\overline{1,K};\,\mathbf{P}_k\in\mathrm{A}_k$$

Therefore, we have obtained an optimal estimate of the model parameters for the observed data. The next step in data processing is to synthesize the model signal using the model parameters and estimate the differences between the model and the observation data for the selected criterion. Therefore, it can be concluded that mathematical model 5 approximates the observation data of the explosive signal with a high degree of confidence, which makes it possible to represent the CIO in the vector space of free model parameters. That is, the dynamics of the vector of the model that reflects this object characterizes the dynamics of changes in the object itself. An effective model for analyzing the spectral characteristics of critical infrastructure objects lying in the seismic and lower part of the acoustic frequency ranges based on monitoring their dynamics is proposed.

Opinions

A new effective model for analyzing the effect of blast waves on the state of CIO, whose natural frequencies lie in the seismic and lower part of the acoustic frequency ranges, based on monitoring their dynamics, is proposed. A new method for identifying the state of such objects is proposed. A non-traditional model of the natural background of the monitored object in the form of a superposition of Berlage pulses is proposed. This model makes it possible to estimate such an important parameter in the description of an object as its quality factor, the dynamics of which can give an idea of its structural changes.

The mathematical model proposed in this work is universal for seismic and acoustic monitoring of critical infrastructure objects based on seismic and acoustic analysis, and makes it possible to conduct seismic and acoustic monitoring of such objects of various physical and geometric characteristics, such as natural objects (bases of building structures of CIO, landslides on which CIO are located, and cracks that affect CIO), building structures of large geometric dimensions with seismic frequency range, up to the components of the object of study of small dimensions with the spectral range of the upper part of the sound range.

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