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A METHOD OF ANALYSIS OF DEPENDENCE BETWEEN SAMPLING RATE OF SIGNAL AND ITS APPROXIMATION USING INTERPOLATION ANALOGUES

Modern digital signal processing is very useful and productive field of IT. Nowadays methods of this field are used in solving of a number of problems in data science, artificial intelligence, economics and other spheres of human activity. As research of last years shows, special interest is concentrated on some groups of methods of digital signal processing, one of which is a group of methods, what are based on Fourier analysis. Especially last research demonstrate an interest to operators, generated by linear summation methods of Fourier series, and its interpolation analogues. This class of methods in context of digital signal processing allows to solve tasks, related with signals approximation, signal filtering and some other aspects of signals analysis. In other hand, some of these methods are well studied in context of Fourier analysis and approximation theory, what allows to make general understanding of possibilities and specifics if its usage in context of digital signal processing. Despite it, there is a number of aspects, specific for digital signal processing, what must be studied but are usually ignored by researchers. One of these aspects is dependence between a sampling rate of considered signal and accuracy of approximation of this signal using interpolation analogue of some operator, generated by linear summation method of Fourier series. This work demonstrates a method of studying this dependence and shows an its usage for interpolation analogues of Fejer operators and Abel-Poisson operators. As experiments show, described method of analysis of dependence between sampling rate of considered signal and interpolation analogues have hidden relation with convergence of operators, which interpolation analogue is used, what allows to predict a sampling rate, what is sufficient to guarantee needed accuracy of approximation of a signal.

Keywords: Fourier analysis, DSP, approximation, linear summation, signals analysis.

Introduction

A usage of interpolation is one of classical techniques in digital signal processing (DSP). Using it we may solve a number of problems like approximation of a digitized signal [1-3], its filtering [4] or studying of extrema of considered digital signal [5]. To solve problems like these we may use different operators, generated by linear summation methods of Fourier series:

$$P_{s_1, s_2, \dots, s_v}(t) = P_{s_1, s_2, \dots, s_v}(f, t) = \int_0^T f(x) K_{s_1, s_2, \dots, s_v} \left(\frac{2\pi(x-t)}{T} \right) dx, \quad (1)$$

where T is a signal duration, $f(t)$ is a function, what describes a considered signal, and its interpolation analogues:

$$\tilde{P}_{m,s_1,s_2,\dots,s_v}(t) = \tilde{P}_{m,s_1,s_2,\dots,s_v}(f,t) = \frac{1}{m+1} \sum_{k=0}^m f\left(\frac{Tk}{m}\right) K_{s_1,s_2,\dots,s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right). \quad (2)$$

Despite a variety of research of properties of (1) and (2), we may see, that there are a number of aspects, what are badly studied but are useful in practice. One of these aspects is a dependence between an accuracy of approximation of signals using (2) and a number of samples m . A main goal of this work is a try of description of this dependence for general case of (2) and demonstration for some cases, when (1) is a Fejer operator. (problem formulation).

Literature review

Usage of interpolation analogues (2) of operators (1), generated by linear summation of Fourier series, is very perspective way to solve different problems of DSP. For example, in [4, 6] authors show, that these operators and its interpolation analogues may be used for filtering of one-dimensional signals. In many other works it as shown, that a possible usage of operators, generated by linear summation of Fourier series, may be useful not only in filtering of signals, but in approximation of signals too. For example, [7, 8] shows this and, in addition, demonstrated some basic fundamental aspects of this way of signal approximation. Some other works show, that in case of digital signals we may use interpolation analogues like (2) to solve some more specific tasks of DSP. For example, in [5] authors shown, that using properties of (1) and (2) may be used for estimation of moments of time, in which studied signal have extremal values, what may be useful in a number of fields, strongly related with data and signal processing. But in many cases we need to study a number of aspects, what are more fundamental, than existing results. One of these aspects is dependence between accuracy of approximation of signals using (2) and sapling rate, described by a number of samples m . This work proposes a method of description of this dependence and demonstrates it for interpolation analogues of Fejer operators [3, 5].

Purpose and objectives of the study

A main purpose of this work is to demonstrate a method of analysis of a change of accuracy of approximation of signals using (2) after increasing of sampling rate of these signals, described by a number of samples m . To achieve this purpose, the following tasks have been set:

1. To describe a change of accuracy of approximation of a signal using (2) when a number of samples of this signal on fixed time interval is increased by 1;
2. To demonstrate this change for interpolation analogues (2) for some signals;
3. To analyze possible hidden dependencies between a change of accuracy of approximation of signals using (2) and properties of operators (1).

Main part

Let consider, that studying signal may be described by function $f = f(t)$, $t \in [0; T]$. To estimate a deviation of interpolation analogue (2) from this function it may be natural to use standard deviation:

$$\varepsilon(m) = \varepsilon(\tilde{P}_{m,s_1,s_2,\dots,s_v}, f) = \int_0^T (\tilde{P}_{m,s_1,s_2,\dots,s_v}(t) - f(t))^2 dt. \quad (3)$$

A metrics (3) as measure of deviation between polynomial (2) and function $f(t)$ is simple to understand and often natural. Despite it, a metrics (3) has one big disadvantage: this metrics don't describe a change of variation between (2) and $f(t)$ after increasing of parameter m . Using this fact, we propose to use other, more complex metrics $\delta(m)$, what may be described by formula:

$$\delta(m) = \delta(\tilde{P}_{m,s_1,s_2,\dots,s_v}, f) = \varepsilon(m+1) - \varepsilon(m). \quad (4)$$

Let study this metrics. To do it, let firstly rewrite (4).

$$\varepsilon(m+1) - \varepsilon(m) =$$

$$\begin{aligned}
 &= \int_0^T (\tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) - f(t))^2 dt - \int_0^T (\tilde{P}_{m, s_1, s_2, \dots, s_v}(t) - f(t))^2 dt = \\
 &= \int_0^T (\tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) - \tilde{P}_{m, s_1, s_2, \dots, s_v}(t)) (\tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) + \tilde{P}_{m, s_1, s_2, \dots, s_v}(t) - 2f(t)) dt. \\
 &\quad \tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) - \tilde{P}_{m, s_1, s_2, \dots, s_v}(t) = \\
 &= \frac{1}{m+2} \sum_{k=0}^{m+1} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) - \\
 &\quad - \frac{1}{m+1} \sum_{k=0}^m f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) = \\
 &= f(T) K_{s_1, s_2, \dots, s_v}\left(2\pi - \frac{2\pi t}{T}\right) + \\
 &+ \sum_{k=0}^m \left(\frac{1}{m+2} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) - \frac{1}{m+1} f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) \right). \\
 &\quad \tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) + \tilde{P}_{m, s_1, s_2, \dots, s_v}(t) = \\
 &= \frac{1}{m+2} \sum_{k=0}^{m+1} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) + \\
 &\quad + \frac{1}{m+1} \sum_{k=0}^m f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) = \\
 &= f(T) K_{s_1, s_2, \dots, s_v}\left(2\pi - \frac{2\pi t}{T}\right) + \\
 &+ \sum_{k=0}^m \left(\frac{1}{m+2} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) + \frac{1}{m+1} f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) \right).
 \end{aligned}$$

If we introduce next notation:

$$\begin{aligned}
 \sum_{k=0}^m \left(\frac{1}{m+2} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) - \frac{1}{m+1} f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) \right) &= S_1(t), \\
 \sum_{k=0}^m \left(\frac{1}{m+2} f\left(\frac{Tk}{m+1}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m+1} - \frac{2\pi t}{T}\right) + \frac{1}{m+1} f\left(\frac{Tk}{m}\right) K_{s_1, s_2, \dots, s_v}\left(\frac{2\pi k}{m} - \frac{2\pi t}{T}\right) \right) &= S_2(t),
 \end{aligned}$$

we will have the next:

$$\begin{aligned}
 \delta(m) &= \\
 &= \int_0^T \left(f(T) K_{s_1, s_2, \dots, s_v}\left(2\pi - \frac{2\pi t}{T}\right) + S_1(t) \right) \left(f(T) K_{s_1, s_2, \dots, s_v}\left(2\pi - \frac{2\pi t}{T}\right) + S_2(t) - 2f(t) \right) dt = \\
 &= f^2(T) \int_0^T K_{s_1, s_2, \dots, s_v}^2\left(2\pi - \frac{2\pi t}{T}\right) dt + \\
 &\quad + f(T) \int_0^T (S_1(t) + S_2(t) - 2f(t)) K_{s_1, s_2, \dots, s_v}\left(2\pi - \frac{2\pi t}{T}\right) dt + \\
 &\quad + \int_0^T S_1(t) (S_2(t) - 2f(t)) dt =
 \end{aligned}$$

$$\begin{aligned}
 &= f^2(T) \int_0^T K_{s_1, s_2, \dots, s_v}^2 \left(2\pi - \frac{2\pi t}{T} \right) dt + \int_0^T S_1(t) (S_2(t) - 2f(t)) dt + \\
 &+ 2f(T) \int_0^T (\tilde{P}_{m+1, s_1, s_2, \dots, s_v}(t) - f(t)) K_{s_1, s_2, \dots, s_v} \left(2\pi - \frac{2\pi t}{T} \right) dt. \tag{5}
 \end{aligned}$$

Now, using (5), we may study a dynamics of change of deviation of (2) from function, what describe a considered signal, for every concrete operator (1).

To demonstrate a usage of (5) let consider, that operator (1) is a Fejer operator:

$$\sigma_n(t) = \sigma_n(f, t) = \frac{1}{n} \int_0^T f(x) \frac{1 - \cos \frac{2\pi n(x-t)}{T}}{1 - \cos \frac{2\pi(x-t)}{T}} dx. \tag{6}$$

Let consider, that studying signal may be described by function

$$f(t) = \sum_{k=1}^{10} \cos kx \sqrt{\ln(\sin kt + (k+1)^2) + k}, \quad t \in [0; 2\pi]. \tag{7}$$

To analyze a metrics (4) in case of approximation (7) using interpolation analogue of Fejer operator (6), let visualize (4) for case of different values of n depends on a number of samples m .

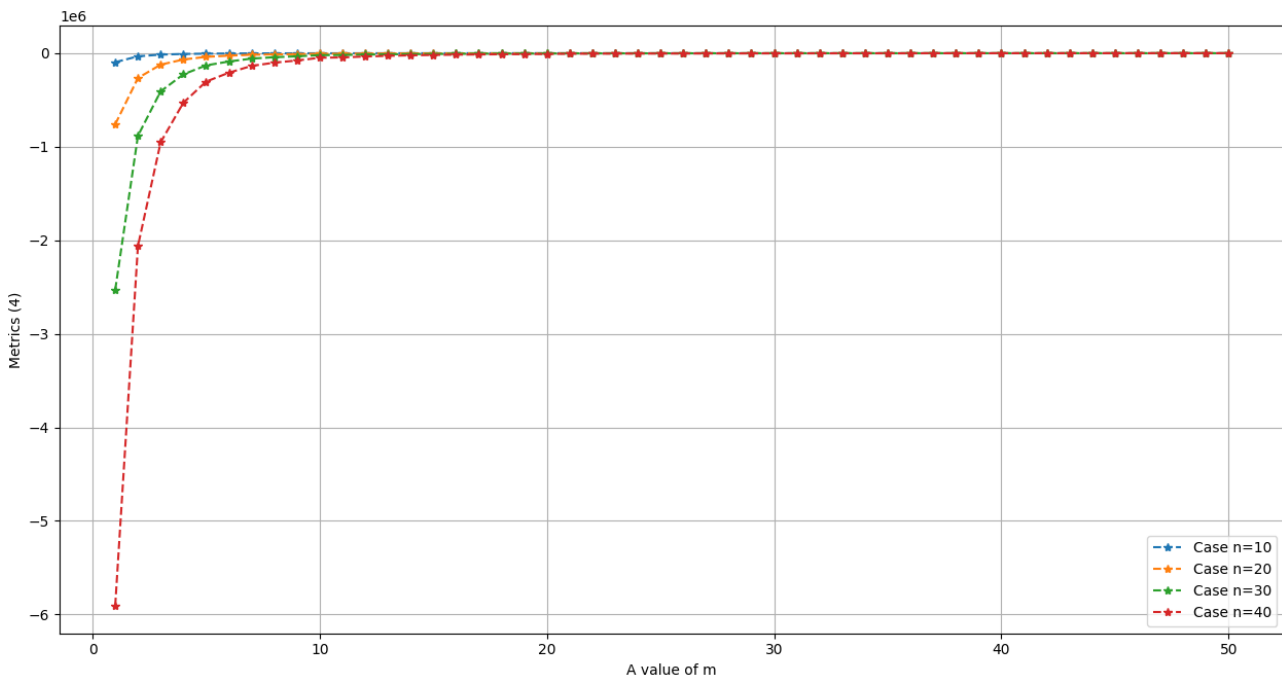


Fig. 1. Dependence of (4) from a number of interpolation nodes m in case of fixed value of parameter n

On fig. 1 we may see, that metrics (4) for interpolation analogues of Fejer operators (6) is going to 0 when a number of samples m on fixed time interval is increasing on 1 for $m \rightarrow \infty$. In context of signal processing it means, that there is a max sampling rate, after increasing of which interpolation analogues of (6) will not give significant increasing of accuracy of signal approximation.

To be sure, that this conclusion is correct, let consider other signal. Let consider a signal, what is described by function:

$$f(t) = e^{-(x-2)^4} \left(\Gamma(\cos(x-1) + 3) \sin(x-1) - \Gamma(\sin 2(x-1) + 3) \cos 3(x-1) \right), \quad t \in [0; 4]. \tag{8}$$

By analogy, let visualize metrics (4) for interpolation analogue of (6) for this signal, described by (8).

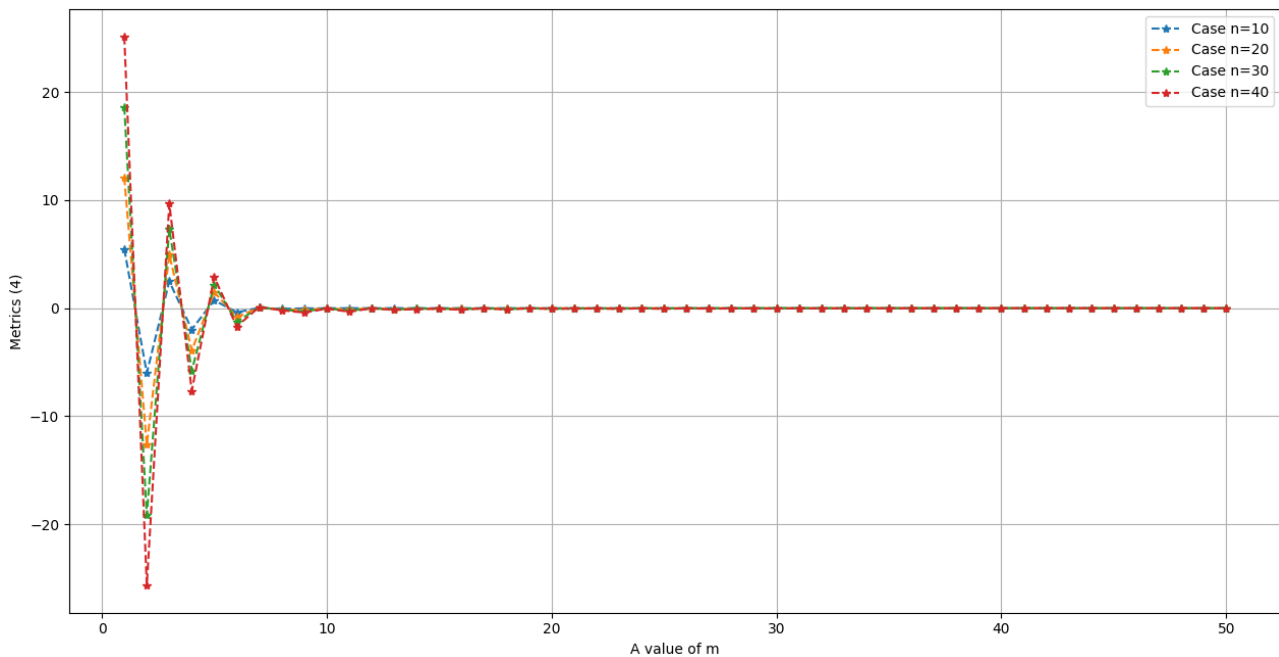


Fig. 2. Dependence of (4) from a number of interpolation nodes m in case of fixed value of parameter n

On fig. 2 we may see similar situation: when greater sampling rate is now, then less significant increasing of accuracy of approximation of the signal we will see after increasing of this sampling rate. All it means, that when approximating a one-dimensional signal using interpolation analogues of Fejer operators (6), there is a max sampling rate of this signal, for increasing of which we will not see a significant increase of accuracy. In general, all these results mean, that using metrics (4) we may study interpolation analogue of operators, generated by linear summation methods of Fourier series, more deeply.

In other hand, from Fourier analysis we know, that if operator (1) is generated by some linear summation method of Fourier series, then for this operator we may say about its convergence. As it known, Fejer operator (6) is convergent. Considering it, it's logical to see on behavior of metrics (4) for other convergent operators of type (1). To do it, let build schedules, analogic to fig.1 and fig. 2, but now let consider an approximation of signals, described by (7) and (8), by interpolation analogues of other convergent operator, for example, Abel-Poisson operator:

$$A_\rho(t) = A_\rho(f, t) = (1 - \rho^2) \int_0^T \frac{f(x)}{1 - 2\rho \cos \frac{2\pi(x-t)}{T} + \rho^2} dt, \quad \rho \in (0, 1]. \quad (9)$$

These schedules will be next (fig. 3-4).

On fig. 3 and fig. 4 we may see same behavior: increasing of sampling rate may give a significant increasing of accuracy of approximation using interpolation analogues of (9) only if a number of samples is low. It means, that if polynomials (2) are interpolation analogue of convergent operators (1), generated by linear summation of Fourier series, increasing of sampling rate of signal may give significant increasing of accuracy of approximation of this signal only if this sampling rate is relatively low, in opposite case increasing of sampling rate is unjustifiably.

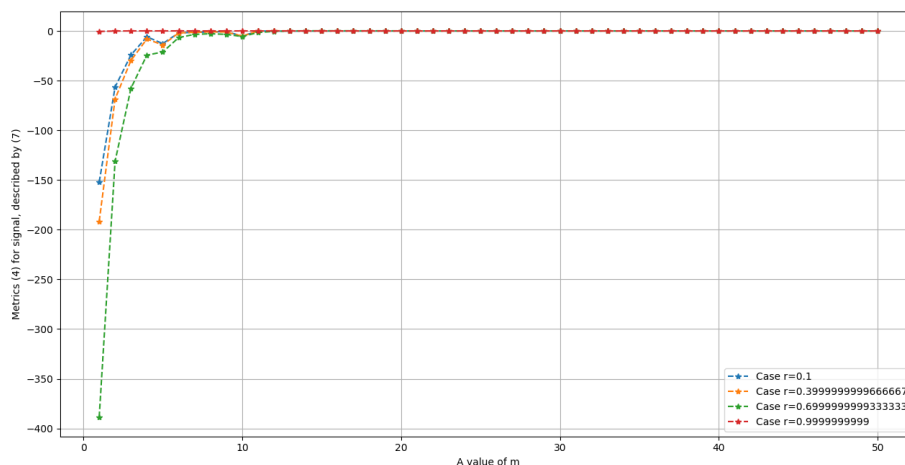


Fig. 3. Dependence of (4) from a number of interpolation nodes m in case of fixed value of parameter ρ

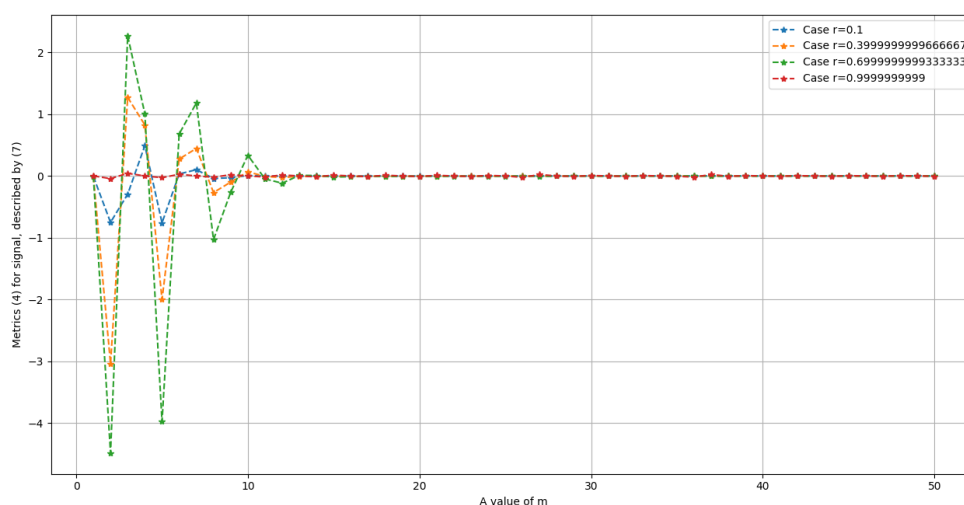


Fig. 4. Dependence of (4) from a number of interpolation nodes m in case of fixed value of parameter ρ

Results

A main result of this work is a description of possible way to study dependence between change of sampling rate of a signal, described by a number of samples m , and accuracy of approximation of this signal using interpolation analogues (2) of operators (1), generated by linear summation of Fourier series. As experiments shows, if an operator (1) is convergent, an absolute value of change (4) of standard deviation between its interpolation analogue and a function, what describes considered function, will go to zero in case of increasing of a sampling rate. Despite this result is purely empirical, this result is consistent with results of previous research, what is described, for example, in [9].

Conclusions

The work demonstrates a method of studying of dependence between a number of samples of one-dimensional signal and approximation of this signal using interpolation analogues (2) of operators (1), what are generated by linear summation of Fourier series. This method is based on usage of metrics (4), what actually describes a change of standard deviation of (2) from function, which describes a considered signal, after increasing of a number of samples in fixed time interval on one.

In the research, a usage of metrics (4) for interpolation analogue of Fejer operator is demonstrated for two cases. Both cases show, that for interpolation analogues of Fejer operator there is some limit value of sampling rate, beyond which an increase of accuracy of approximation of signal will not significant. It means, that if we try to approximate a digital signal using interpolation analogue

of Fejer operator, there is no sense to give very high sampling rate for sampling considered signal, because it will give an unnecessarily large number of calculations. As similar research of interpolation analogues of other operators, given by linear summation of Fourier series, shows, probably we may have analogic behavior, but to approve it, it would be good to do additional research. Results of this research may be useful in tasks, what propose a work with DSP [10-12].

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